What force makes the current flow?

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1. Electromagnetic analyse

A current is a flow of electric charges. In a conductor, moving charges are slowed down by friction that we call resistance. Thus, an external force must act on the free charges to maintain them in move. Otherwise, the current would stop. In a coil plunged in variable magnetic field, an induced current occurs indicating the existence of a external force on the free charges. If the coil is open, a voltage will appear between the 2 terminals revealing the electric field and the force exerted on the free charges within the conductor. This force is applied by the electric field that the variable magnetic field induces, which is governed by the 3rd Maxwell's equation:

Eq. 1 $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

We used to study the induction phenomenon by the electromagnetic fields way. Here, I propose to study the force exerted on the charges. Figure 1 shows a ring of radius R_a plunged in the variable magnetic field of the current I. The coil and the ring are parallel to each other. The reference frame is cylindrical and has its origin at the common centre of the ring and coil.

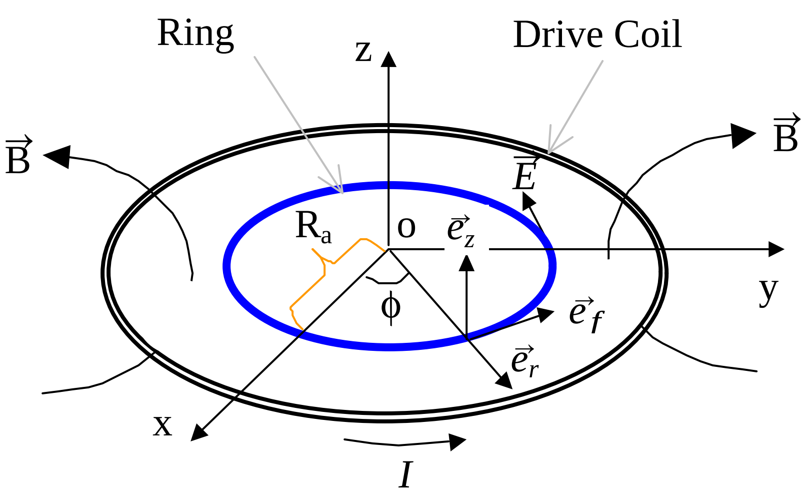

Figure 1 Force on a ring

Naturally, a charge q receives a force $\vec{F} = q\vec{E}$ from the electric field that the magnetic field induces. If the charge is uniformly distributed on the ring, for example in a conductor, this force makes a the torque with respect to the centre o, which is the integral of the elementary moment $\vec{r} \times d\vec{F}$ over the ring. The torque is in z direction:

Eq. 2 $$M_{ring} = \vec{e}_z \cdot \oint \vec{r} \times d\vec{F} = \oint \left(d\vec{F} \cdot \vec{e}_z \times \vec{r} \right)$$

The elementary force $d\vec{F}$ is exerted on a infinitesimal length of the ring $dl=R_a\, d\phi$. The charge of this segment is "line density"*length: $dq=\rho_l\, R_a\, d\phi$. By replacing $d\vec{F} = dq\vec{E}$, in Eq. 2, the torque becomes:

Eq. 3 $$M_{ring} = \oint \left(\rho_l R_a d\phi\, \vec{E} \right) \cdot \left(R_a \vec{e}_\phi \right) = R_a\, \rho_l \oint \vec{E} \cdot d\vec{l}$$

According to the Stokes' theorem, the circulation of $\vec{E}$ is equal to the flux of $\nabla \times \vec{E}$ that passes through the area bounded by the ring. By applying the 3rd Maxwell equation (Eq. 1), the torque reveals to be proportional to the variation rate of the magnetic flux $\int_{area} \vec{B} d\vec{s}$ that is equal to "self inductance"*I:

Eq. 4 $$\int_{area} \vec{B} d\vec{s} = LI$$

Eq. 5 $$M_{ring} = R_a\, \rho_l \int_{area} \nabla \times \vec{E} \cdot d\vec{s} = -R_a\, \rho_l \frac{\partial}{\partial t} \left(\int_{area} \vec{B} d\vec{s} \right)$$

Eq. 6 $$M_{ring} = -R_a\, \rho_l L \frac{dI}{dt}$$

In consequence, the torque is proportional to $\frac{dI}{dt}$ .This torque is of course non-null because a current occurs in a conductive ring. Its sign is negative in accordance with the Lenz' law. M_{ring} is exerted on positive charges. The torque on negative charges has opposite sign. In a neutral ring, there are as many positive charges as negative charges. The torques on them cancel out each other so that the resultant torque is 0. Finally, the ring would not turn under the torque of Eq. 6.

But, what would occur if there were more positive charge than negatives? Would the ring spin? Let us see this case with a charged dielectric ring. In such matter, the charges are not free to move so that the force they receive are transmitted to the ring. Seemingly, a charged dielectric ring would spin. If the ring is fixed, the torque is measurable. Nevertheless, no such torque has ever been measured. Why?

2. Mechanical analyse

Let us analyse this torque by the mechanical way (see Figure 2). On a point of the ring, P_r, the force on it is $d\vec{F}=dq\,\vec{E}$. The electric field $\vec{E}$ is the integral of the contribution

$d\vec{E}$ of all points of the coil P_c : $\vec{E} = \int_{coil} d\vec{E}$. The torque on the ring is the integral on the elementary moment : $d\vec{M}_{ring} = \overrightarrow{oP_r} \times d\vec{F}$:

Eq. 7 $\vec{M}_{ring} = \int_{ring} \overrightarrow{oP_r} \times d\vec{F} = \int_{ring} \overrightarrow{oP_r} \times \left(dq\vec{E}\right) = \int_{ring} \overrightarrow{oP_r} \times \left(dq \int_{coil} d\vec{E}\right) = \int_{ring} \left(\int_{coil} \overrightarrow{oP_r} \times dqd\vec{E}\right)$

Let $d^2\vec{F} = dq\, d\,\vec{E}$ be the elementary force between a point of the ring P_r and one of the coil P_c (see Figure 2), $d^2\vec{M}_{ring} = \overrightarrow{oP_r} \times d^2\vec{F}$ be the moment of this force to the centre of the ring. The torque on the ring is the double integral of $d^2\vec{M}_{ring}$:

Eq. 8 $$\vec{M}_{ring} = \int_{ring} \int_{coil} \overrightarrow{oP_r} \times d^2\vec{F} = \int_{ring} \int_{coil} d^2\vec{M}_{ring}$$

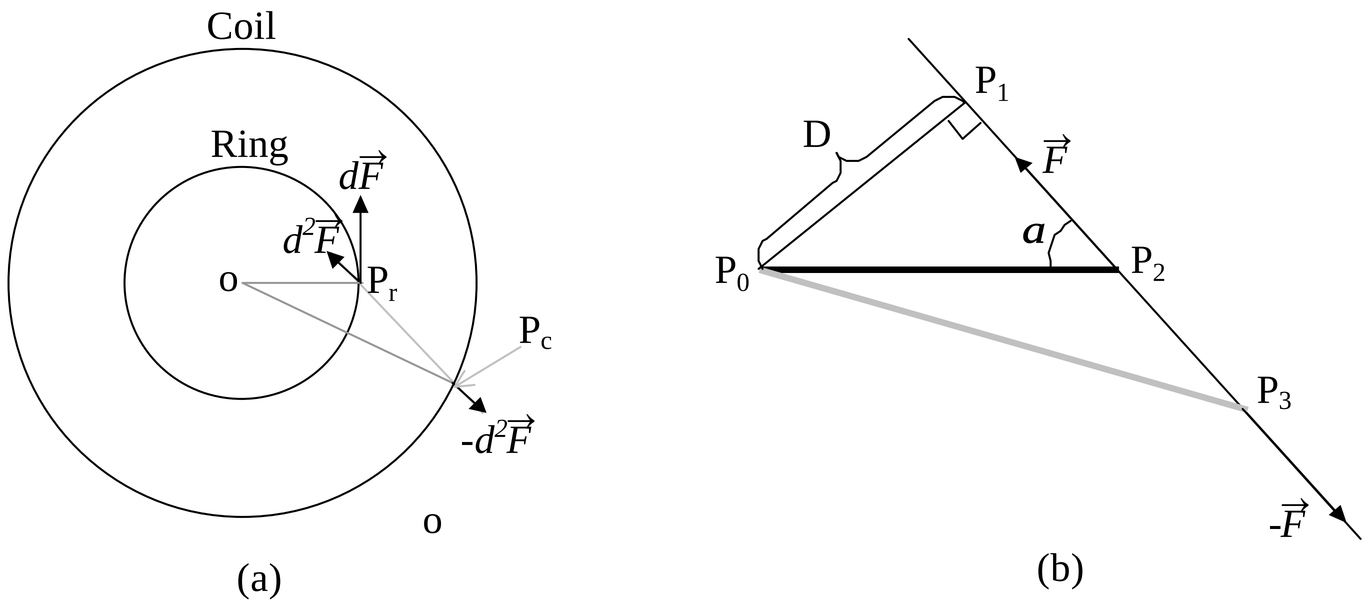

Figure 2 Mechanical torque

In Figure 2 (b), a force $\vec{F}$ is applied to the point P_2. To the point P_0, the arm of the force is $\overrightarrow{P_0P_2}$. The moment of this force is the cross product between the arm vector and the force vector:

Eq. 9 $$\vec{M} = \overrightarrow{P_0P_2} \times \vec{F} = P_0P_2 \cdot F \sin a = D \cdot F$$

If the force $\vec{F}$ is applied to another point of the straight line that is determined by the point P_2 and the direction of $\vec{F}$, for example P_1 or P_3 , its moment to P_0 is also equal to $D*F$. If the force on P_3 is equal to $-\vec{F}$, its moment to P_0 is $-D*F$. Thus, for any pair of points P_2, P_3 on which are applied respectively the action and reaction forces $\vec{F}$ and $-\vec{F}$, the moment of $\vec{F}$ to P_0 is $D*F$ and that of $-\vec{F}$ is $-D*F$ because the force $\vec{F}$ is always parallel to $\overrightarrow{P_2P_3}$.

For the pair of points P_r of the ring and P_c of the coil (see Figure 2 (a)), the action force on P_r is $d^2\vec{F}$ and the reaction force on P_c is $-d^2\vec{F}$ in accordance with the 3rd Newton's law. So, the moment on the coil to its centre o must be the double integral of:

Eq. 10 $$d^2\vec{M}_{coil} = \overrightarrow{oP_c} \times \left(-d^2\vec{F}\right) = -\overrightarrow{oP_r} \times d^2\vec{F} = d^2\vec{M}_{ring} :$$

Eq. 11 $$\vec{M}_{coil} = \int_{ring}\int_{coil} -d^2\vec{M}_{ring} = -\vec{M}_{ring}$$

Does the coil receive really a force from the ring? The electrostatic force on the coil is 0 because it is neutral. The Lorentz' force on the coil that carries a current is 0 because the ring is fixed and does not generate any magnetic field. So, the ring does not apply any reaction force on the coil. In consequence, the torque on the coil is 0:

Eq. 12 $$\vec{M}_{coil} = 0$$

According to the 3rd Newton's law (see Eq. 11), the torque on the ring would be 0:

Eq. 13 $$\vec{M}_{ring} = 0$$

Then, what force would make the current flow in a conductive ring?

3. Energy analyse

Now, let us spin the ring at a constant speed ω. The torque M_{ring} will do a mechanical power:

Eq. 14 $$P_{mec} = \omega M_{ring} = -\omega R_a \rho_l L \frac{dI}{dt}$$

This power is provided by the coil because M_{ring} results from the current. The electric power of the coil is UI where U is the induced voltage within the coil. According to the Faraday's law, U is equal to the variation rate of the total magnetic flux :

Eq. 15 $$U = -\frac{d\left(B_{ring} + B_{coil}\right)}{dt}$$

As the angular speed of the ring is constant, B_{ring} is constant: $\frac{dB_{ring}}{dt} = 0$. The flux due to the coil is proportional to the current: $B_{coil} = \int_{area} \vec{B}d\vec{s} = LI$ (see Eq. 4) so that the electric power is finally equal to the magnetic power that is stored in the coil:

Eq. 16 $$P_e = UI = -L\frac{dI}{dt}I = -\frac{L}{2}\frac{dI^2}{dt} = P_{mag}$$

The energy that the coil provides to the ring is the energy it receives from the current source minus the magnetic energy it stores: P_e-P_{mag}=0. So, the coil does not give energy to the ring. In consequence: the ring does not receive energy: $\omega M_{ring}=0$. Or :

Eq. 17 $$M_{ring}=0$$

4. Electrostatic force analyse

The intermediary that transports the force between the coil and the ring is the induced electric field. Let us suppose that the induced electric field may hold force as more than 1 person suggest. The balance of the torques on the ring, on the coil and held by the field is null :

Eq. 18 $$\vec{M}_{ring} + \vec{M}_{coil} + \vec{M}_{field} = 0$$

As $\vec{M}_{coil} = 0$ (see Eq. 12), the torque held by the field is:

Eq. 19 $$\vec{M}_{field} = -\vec{M}_{ring}$$

The torque on the ring depends on its charge (see Eq. 3). If the charge on the ring reverses its sign, $\boldsymbol{r}^{-}=-\boldsymbol{r}_l$, the torque on the ring reverses its sign for the same induced field:

Eq. 20 $$\vec{M}^{-}_{ring} = R_a \boldsymbol{r}^{-}\oint \vec{E}\cdot d\vec{l} = -R_a \boldsymbol{r}_l \oint \vec{E}\cdot d\vec{l} = -\vec{M}_{ring}$$

If we reverse the sign of the charge and maintain the current in the coil, the induced field stays unchanged. The torque held by the induced field are:

For positive charge on the ring $\quad \vec{M}_{field} = -\vec{M}_{ring}$

For negative charge on the ring $\quad \vec{M}^{-}_{field} = -\vec{M}^{-}_{ring} = \vec{M}_{ring}$

As the induced field stays unchanged, the torque held by the induced field is unchanged:

Eq. 21 $$\vec{M}^{-}_{field} = \vec{M}_{field}$$

So, the torque held by the induced electric field is equal to its opposite, i.e. 0:

Eq. 22 $$\vec{M}_{field} = -\vec{M}_{field} \Rightarrow \vec{M}_{field} = 0$$

In consequence, the torque on the ring is 0 too:

Eq. 23 $$\vec{M}_{ring} = 0$$

5. Conclusion

Contrary to the Mechanical analyse, the Energy analyse and the Electrostatic force analyse, there must be a non null torque on the ring because a current will flow if the ring is conductive. This current is made by the force the electrons receive. According to my last study, this non null torque is counter-balanced by a hidden electric force that makes the 3rd Newton's law and the energy conservation respected. The classical electromagnetic theory concluded a null torque over the coil because this force was not taken into account. If we measure the torque, we will see that it is non null and even stronger than the prediction of the 3rd Maxwell's equation. The experiment "Fame bringing experiment" will definitively decide about this torque and the validity of the 3rd Maxwell's equation.

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24 May 2002	Creation
28-May-02	Addition of Mechanical analyse
31 May. 02	Addition of the energy analyse and the experiment description
2 June 2002	Addition of Coulomb force analyse
6-Jun-02	Prediction of the experiment
6-Jun-02	Lastly, in the experiment, the ring will spin and the torque is bigger than that of the 3rd Maxwell's equation
15 June 2002	The experiment is put in <<Fame bringing experiment>>